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CHANGING STRUCTURES AT ELECTRICITY MARKETS: MODELLING SPOT PRICES USING TIME-VARYING STABLE CARMA MODELS

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Abstract: Electricity markets are affected by rapidly changing structures, in particular due to the increasing share of renewable energies. Hence, the use of stationary time series models for modelling spot prices becomes more and more questionable. As a step towards the tractability of non-stationary time series we introduce in this paper a new class of stochastic processes which can be used in situations where the time series data at hand exhibit a non-stationary behaviour. These processes behave locally like classical α-stable processes although the parameters can vary over time. We illustrate the estimation of such processes using a straightforward maximum likelihood approach. Moreover, we show how the model can be applied to electricity spot prices. The approach of the paper can be transferred to other areas of applications and, therefore, should open the door to a new way of handling real-life phenomena with nonstationary behavior.

Keywords: Electricity prices-independent increment process-non-stationary process-time-varying parameters.

1. Introduction

Stationary time series models are used in statistics to describe phenomena and their timely development in many areas of our daily life. The idea behind these models is that there is no structural change or break of these phenomena over time, at least after disentangling deterministic patterns as trends or seasonality from the stochastic component. However, in a world of rapidly changing structures in all areas of business and economics, the use of such stationary time series models becomes more and more questionable in many applications and might lead to a systematic underestimation of risk.

Electricity markets which are - compared to classical financial markets - relatively young. The trading rules at energy exchanges are adapted regularly and are subject to change as well as the strategies of the traders, so that it can hardly be assumed that the observed stochastic price processes are stationary. The development of the markets as well as the increasing production of renewable energies indeed seem to create a systematic change of the behaviour of the electricity prices. For general introductions to statistical modeling of electricity markets see, for instance, Benth *et al.* (2008) and Weron (2006).

In view of these facts, we define and investigate in this paper a new class of processes, where the parameters of the classical α-stable process can vary over time, i.e. they follow deterministic functions which possibly incorporate an, in principle, arbitrary number of unknown parameters. The α-stable processes are attractive in many applications, since on the one hand they belong to the class of Lévy processes and on the other hand their increments allow for more extreme events, as the parameter $\alpha \in (0, 2]$ decreases from its upper limit 2. In addition, when one uses the parameter $\alpha = 2$, one gets back to the widely used Brownian motion. A broad overview over the theory of classical α-stable processes can be found in Samorodnitsky and Taqqu (1994).

In general, the treatment of non-stationary time series is gaining more and more attention, and different approaches can be found in the literature. The idea to have locally approximately a stationary process was already used in the theory of processes with evolutionary spectra, cf. Priestley (1965), where the processes are defined via a time varying spectral representation. Another approach is taken in Dahlhaus (2012) and Dahlhaus *et al.* (2017) where locally stationary processes are defined using infill-asymptotics from nonparametric statistics. This idea is, however, quite different from our approach, since locally stationary processes are based on discrete-time models as, e.g., classical autoregressive models like AR(1). Furthermore, some literature employs classical time series models, but replaces the constant parameters by time-varying versions. A recent paper of this type is Müller and Uhl (2021) which define time-varying stochastic volatility models with α-stable innovations, in the classical discrete time setup. Our approach uses, in contrast to all those other ideas, the theory of additive processes, *i.e.* of continuous-time processes with independent, but not (necessarily) stationary increments. Hence, we can employ a much broader class of processes than the widely used Lévy processes which necessarily have stationary increments. Some theoretical basics of additive processes in general can be found, *e.g.*, in Sato (1999).

The paper is organized as follows. In Section 2 we define the class of the nearly α-stable additive (NASA) processes and prove existence. In Section 3 we simulate NASA processes and investigate the quality of maximum likelihood estimates of the parameters. Moreover, we replace the α-stable process from Benth *et al.* (2014) and Müller and Seibert (2019) by a NASA process and check the quality of the Bayesian posterior mean estimates in a simulation study. In Section 4 we apply the new model to electricity data from the EEX. Section 5 concludes.

2. Nearly α-stable Additive Processes

In this section we merge the concept of time-varying parameters with α-stable processes which have been used in Benth *et al.* (2014) and Müller and Seibert (2019) for modelling electricity spot prices. The idea of these papers is to use a continuous-time autoregressive moving-average (CARMA) process to describe the behaviour of electricity prices. The CARMA process is driven by an α-stable Lévy process, to capture the price spikes.

In order to gain exibility about the spike distribution we want to allow for time varying parameters in the α-stable process. Whereas this formally can be done quite easily, we have to check whether the resulting process is a mathematically reasonable object. In particular, we need the resulting process to be *e.g.* a semimartigale, since we want it to drive the CARMA process, via stochastic integration.

Hence, in the following, we care about a mathematically sound definition of α-stable processes with time-varying parameters and the desired semimartingale property.

Belonging to the class of classical Lévy processes, also α-stable processes are characterized by the Lévy triplet (g, σ, ν). This triplet shows that a Lévy process can be interpreted as consisting of three independent components: a linear drift, a Brownian motion, and a Lévy jump process. In case of the α-stable process the Brownian component vanishes and the Borel measure is specified as

$$\nu(x) = (c_{-}1_{x<0} + c_{+}1_{x>0})/|x|^{1+\alpha},$$

for parameters $c_{-}, c_{+} \geq 0$ and $\alpha \in [0; 2]$. The limit case $\alpha = 2$ corresponds to the Brownian motion.

In order to give up the property of Lévy processes of having stationary increments, we replace the parameters α, c_{+}, c_{-} and g by time-dependent variables $\boldsymbol{\alpha}(t)$, $\boldsymbol{c}_{+}(t)$, $\boldsymbol{c}_{-}(t)$ and $\boldsymbol{g}(t)$, respectively. This way we derive in the following a fairly general subclass of additive processes in law, by applying a time-space approach.

Let $\boldsymbol{\alpha} : [0, \infty) \to (0, 2)$ and $\boldsymbol{c}_{\pm} : [0, \infty) \to [0, \infty)$ be Borel functions. Moreover, introduce functions $\boldsymbol{c}_{*} := \boldsymbol{c}_{-} + \boldsymbol{c}_{+} : [0, \infty) \to [0, \infty)$ and $\boldsymbol{c}_{\Delta} := \boldsymbol{c}_{+} - \boldsymbol{c}_{-} : [0, \infty) \to \mathbb{R}$, and set $\boldsymbol{c}(t, x) := \boldsymbol{c}_{-}(t)1_{x<0} + \boldsymbol{c}_{+}(t)1_{x>0}$.

Given the functions $\boldsymbol{\alpha}$ and $\boldsymbol{c}$, introduce now another Borel function $\nu : [0, \infty) \times \mathbb{R}_{*} \to [0, \infty)$ by setting

$$\nu(t, x) := \boldsymbol{c}(t, x)/|x|^{1+\boldsymbol{\alpha}(t)},\ t \geq 0,\ x \in \mathbb{R}_{*}.$$

To this function we associate a nonnegative Borel measure ν on $[0, \infty) \times \mathbb{R}_{*}$ in time-space by setting

$$\boldsymbol{\mathcal{V}}(A) := \iint_A \nu(t,x)\,dt\,dx \quad \text{for Borel } A \subseteq [0, \infty) \times \mathbb{R}_{*}.$$

For $t > 0$, let $\mathcal{V}_t$ be the time-projected measure associated to $\boldsymbol{\mathcal{V}}$, *i.e.* the Borel measure on $\mathbb{R}_{*}$ dened by

$$\mathcal{V}_t(B) := \mathcal{V}([0, t] \times B) = \iint_{[0,t]\times B} \nu(s,x)\,ds\,dx \quad \text{for Borel } B \subseteq \mathbb{R}_{*}.$$

Based on this time-projected measure and a given deterministic function $\boldsymbol{g} : [0, \infty) \to \mathbb{R}$, we can now state necessary and sufficient conditions in terms of $\boldsymbol{g}$, $\boldsymbol{\alpha}$ and $\boldsymbol{c}_{*}$ to ensure the existence of an additive process L associated to pairs $(\boldsymbol{g}, \mathcal{V})$.

Theorem 1: Suppose $t \to g(t)$ is continuous on $[0, \infty)$ with $g(0) = 0$. There exists (uniquely up to identity in law) an additive process $(L_t)_{t\geq 0}$ with Lévy triplet $(g(t), 0, \nu_t)_{t\geq 0}$ if and only if

(A). $(c_- + c_+)/a$ and $(c_- + c_+)/(2 - a)$ are locally (Lebesgue) integrable on $[0, \infty)$.

Proof: It holds that, for $t \geq 0$, and with $c_* = c_- + c_+$

$$\mathcal{V}_t((-\infty, 1)\cup(1, \infty)) = \int_{[0,t]}\int_1^\infty \frac{c_*(s)}{x^{1+a(s)}}\,dx\,ds = \int_{[0,t]} \frac{c_*(s)}{a(s)}\,ds,$$

and, also,

$$\int_{0<|x|\leq 1} x^2 \mathcal{V}_t(dx) = \int_{[0,t]}\int_{0<|x|\leq 1} \frac{c_*(s)}{x^{a(s)-1}}\,dx\,ds = \int_{[0,t]} \frac{c_*(s)}{2-a(s)}\,ds.$$

Now use Sato (1999), Thm. 9.8 and Rem. 9.9.

We are now prepared to introduce the notion of nearly alpha stable additive (NASA) processes.

Definition 1: We write $L \sim \text{NASA}(\alpha, \boldsymbol{c}, \boldsymbol{g})$, if, simultaneously, $\boldsymbol{g}$ is continuous with $\boldsymbol{g}(0) = 0$, $\boldsymbol{c}_\pm/\alpha$ and $\boldsymbol{c}_\pm(2 - \alpha)$ are locally (Lebesgue) integrable on $[0, \infty)$ and $L = (L_t)_{t\geq 0}$ is the associated additive process in Theorem 1.

If the associated additive process $L \sim \text{NASA}(\alpha, \boldsymbol{c}, \boldsymbol{g})$ in Theorem 1 exists, then the characteristic exponent of its increments is, for $t > s > 0$, obviously given by

$$\Psi_{L_t - L_s}(\theta) = i\theta(\boldsymbol{g}(t) - \boldsymbol{g}(s)) + \iint_{(s,t]\times\mathbb{R}} (e^{i\theta x} - 1 - ix\theta 1_{0<|x|\leq 1})\nu(u,x)\,dx\,du, \quad \infty \in \mathbb{R}_*. \tag{1}$$

Moreover, matching the characteristic functions, one can easily show that the class of strictly α-stable Lévy processes is a subclass of the NASA-processes. More precisely, if $t \to \boldsymbol{g}(t) = t\boldsymbol{g}(1)$ is a linear function, while both $\alpha(t) \equiv \alpha \in (0, 2)$ and $c_\pm(t) \equiv c_\pm \geq 0$ are constant functions with $c_* \neq 0$, then the associated $\text{NASA}(\alpha, \boldsymbol{c}, \boldsymbol{g})$-process L is an α-stable Lévy process.

In view of many theorems which hold for semimartingales we now care about the question whether NASA processes are semimartingales.

Theorem 2: Assume (A), and suppose $\boldsymbol{g}$ is continuous with $\boldsymbol{g}(0) = 0$. The process $L \sim \text{NASA}(\alpha, \boldsymbol{c}, \boldsymbol{g})$ is a semimartingale if and only if $\boldsymbol{g}$ is locally of finite variation.

Proof: Use Jacod and Shiryaev (2003), Thm. II.4.14.

In particular, this property guarantees that we can integrate w.r.t. NASA processes. This feature is important when we replace the driving Lévy process in CARMA processes by a NASA process to make the model more flexible. Note that there are different parametrizations of the α-stable process which are commonly used. Whereas for our theoretical investigations the $(\alpha, \boldsymbol{c}, \boldsymbol{g})$-parametrization used above is convenient, the socalled $(\alpha, \gamma, \beta, \mu)$-parametrization is better interpretable in practical applications. Here, γ denotes the scale parameter, β the skewness parameter, μ the location parameter. Both parametrizations can be transformed into each other, and the shown properties of NASA-processes are, of course, independent of the used parametrization. Hence, in Sections 3 and 4, we will exclusively use the $(\alpha, \gamma, \beta, \mu)$-parametrization, *i.e.* using functional parameters β, γ, μ instead of c_+, c_-, g while α remains unchanged. This enables us to interpret the results more easily regarding skewness and scaling, by looking at the parameters β and γ, respectively.

3. Simulation and Estimation of NASA Processes

We first illustrate how NASA processes behave. Moreover, we are interested in the quality of the estimates, when the time-varying parameters are reestimated from simulated data using the maximum likelihood method.

In this section we assume that

$$\alpha(t) = \min(\alpha_{start} + t(\alpha_{end} - \alpha_{start})/10000, \alpha_{end}),$$
$$\beta(t) = \max((\beta_{start} + t(\beta_{end} - \beta_{start})/10000, \beta_{end}),$$

and that $\gamma \equiv 1$ and $\mu \equiv 0$.

These functions are locally Lipschitz continuous so that the associated NASA process exists as a well-defined semimartingale.

First, we approximate a NASA process based on the functions above by simulating independent increments for the intervals $[t, t + \Delta t]$, $t = 0, \Delta t, 2\Delta t \ldots, 99999\Delta t$ for $\Delta t = 0{:}01$ from corresponding α-stable distributions with parameters $\alpha(t + \Delta t/2)$, $\beta(t + \Delta t/2)$, $\gamma \equiv 1$, $\mu \equiv 0$. Figure 1 shows the result for $\alpha_{start} = 1.5$, $\alpha_{end} = 1.8$, $\beta_{start} = 0.8$, $\beta_{end} = -0.8$. From the lower plot it is clearly to see that the spikes are much more pronounced at the beginning than at the last part of the series, and that the rst period shows mainly upward spikes, the middle period upward spikes as well as downwards spikes, whereas the last period shows mainly downward spikes.

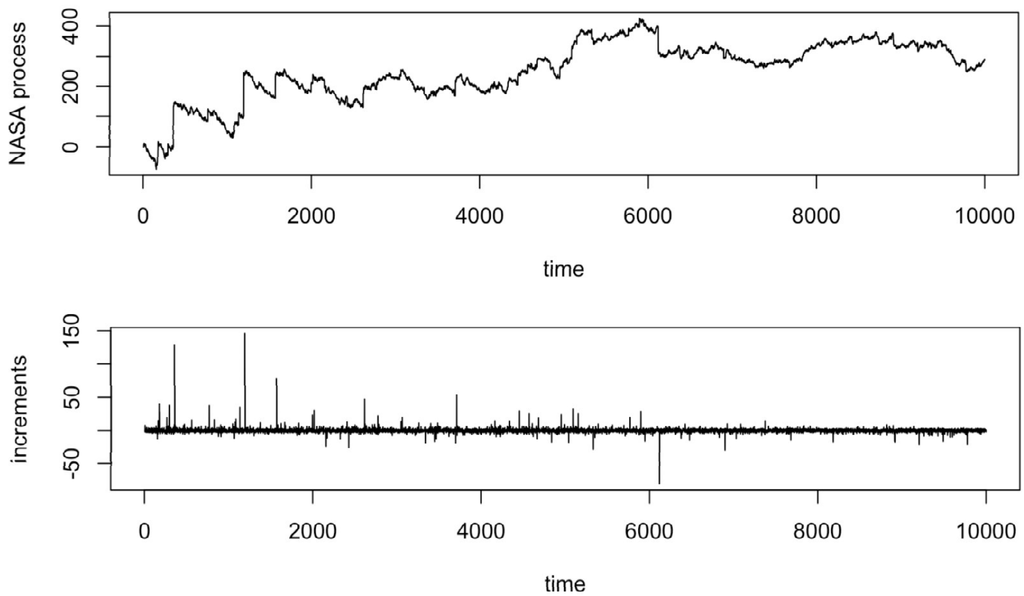

Figure 1: Simulation of NASA process with time-varying stability and skewness functions.

Table 1: Means and standard deviations over 1000 ML estimates for the unknown parameters for 1000 simulated data sets

Parameter	*True*	*Mean*	*sd*
α_{start}	1.50	1.5039	0.0262
α_{end}	1.80	1.8009	0.0263
α_{start}	0.20	0.2046	0.0440
α_{end}	–0.20	–0.2047	0.0569
γ	1.00	1.0003	0.0097

Next we try to estimate the parameters of a NASA process by maximum likelihood. To this end, we use the package stabledist of the statistics software *R*, where an approximation of the stable densities is available. The functions α and β are parameterized as before by

$$\alpha(t) = \alpha_{start} + t(\alpha_{end} - \alpha_{start})/10000, \ \beta(t) = \beta_{start} + t(\beta_{end} - \beta_{start})/10000.$$

In addition, we assume $\gamma \equiv \gamma$ to be time-invariant and unknown. To check the quality of this straight-forward estimation procedure we simulate 1000 data sets, each of length 10000, using the parameter values given in Table 1, and re-estimate the five parameters from the data by ML. The means and standard deviations over the 1000 ML estimates are shown in Table 1.

Obviously, we can estimate the unknown parameters of the NASA process quite well. The largest standard deviation is found for end, which can easily be explained by the fact, that the parameter end is 1.8, *i.e.* relatively close to 2. For a stability parameter $\alpha = 2$, however, the skewness parameter loses its meaning completely, *i.e.* it cannot be estimated at all.

4. Application to Electricity Spot Price Data

Possible applications of the NASA processes can be found in situations, where Lévy processes are involved to model data which exhibits an unstationary behaviour. As an example, we focus here on electricity spot and futures prices.

The electricity markets show specific features which differ from the behaviour of time series at classical financial markets. The most important features of electricity prices are: extreme spikes, possibly negative prices, and quick mean reversion. For all of these features there is (basically) one reason: the balance between demand and supply which is extremely sensitive due to the fact that energy cannot be stored (at least currently not to a larger extent). Whereas positive spikes occur when the supply suddenly breaks down (*e.g.* by a failure of a power plant), negative prices occur when the demand is unexpectedly much lower than the current electricity production. After fixing an imbalance problem, the price usually quickly returns to a mean level determined by a well-working supply and a demand as predicted.

In order to describe electricity prices several stochastic models have been developed. Benth *et al.* (2014) use a three-component additive model, consisting of a deterministic trend and seasonality function, a CARMA process driven by an α-stable Lévy process for the spikes, and an additional NIG process for more flexibility in the long-term behaviour. Using the same model (with a more sophisticated seasonality function), Müller and Seibert (2019) developed a Bayesian estimation procedure. In the following, we use this estimation algorithm, since it can easily be adapted for our situation, and try to find out whether one should replace the driving α-stable (Lévy) process in the electricity price model by a NASA process. While referring to the two cited papers for the details, we will now briefly summarize model specications for the reader's convenience.

4.1 Electricity Spot and Futures Price Model

In Benth *et al.* (2014), the electricity spot price S_t is modelled on a continuous time scale and is decomposed into the sum

$$S_t = \Lambda_t + Y_t + Z_t,$$

where Λ_t accounts for all trend and seasonal components, Y_t is a CARMA(2,1) process, *i.e.* a continous-time ARMA process, which accounts for the large price spikes present in electricity spot price data and which is driven by a Lévy process L_t, and Z_t is a Lévy process modelling long-run (but not seasonal) deviations from the mean price.

The trend and seasonality function writes

$$\Lambda_t = \theta_0 + \frac{\theta_{\text{trend}} t}{365.25} + \Lambda_{\text{week}}(t) + \Lambda_{\text{seasons}}(t),$$

where

$$\Lambda_{\text{week}}(t) = [\theta_{Tu}\mathbb{I}_{Tu}(t) + \dots + \theta_{Su}\mathbb{I}_{Su}(t)]\,(1 - \mathbb{I}_{ho}(t)) + \theta_{ho}\mathbb{I}_{ho}(t)$$

$$\Lambda_{\text{seasons}}(t) = \sum_{n=1}^{2}\left[c_n \cos\left(\frac{2\pi tn}{365.25}\right) + s_n \sin\left(\frac{2\pi tn}{365.25}\right)\right]$$

$$+ \theta_{\text{summer}}\mathbb{I}_{[\text{Jul 29–Aug 21}]}(t) + \theta_{\text{Dec}}\mathbb{I}_{[\text{Dec 24–31}]}(t) + \theta_{\text{Jan}}\mathbb{I}_{[\text{Jan 1–6}]}(t).$$

Here, the functions $\mathbb{I}_{.}(\cdot)$ denote indicator functions for the weekdays Tuesday to Sunday, for public holidays, and for special periods with lower electricity consumption in the summer and around Christmas. Hence, altogether, the trend and seasonality function Λt contains 16 parameters.

The Lévy process (Z_t) is specied as an NIG process with the parameters $\alpha_{\text{NIG}} > 0$ (tail heaviness), $\beta_{\text{NIG}} \in [-\alpha_{\text{NIG}}, \alpha_{\text{NIG}}]$ (skewness), $\delta_{\text{NIG}} > 0$ (scale) and μ_{NIG} (shift). In order to avoid an identication problem with the trend function, μ_{NIG} is fixed at $-\delta_{\text{NIG}}\beta_{\text{NIG}}/\sqrt{\alpha^2_{\text{NIG}} - \beta^2_{\text{NIG}}}$, and only three parameters have to be estimated for the process (Z_t).

To disentangle the sum of (Y_t) and (Z_t), futures prices are used. Theoretical considerations using general arbitrage theory lead to the formula

$$\hat{Z}_t \approx \frac{1}{|\{u \geq 40\}|}\sum_{u \geq 40}\left[F(t,u) - \frac{1}{T_2 - T_1}\int_{T_1}^{T_2} \Lambda_\tau d\tau\right] - \frac{\bar{u}\theta_Q}{365.25} - C, \tag{2}$$

where $F(t, u)$ is the futures price at time t with time to maturity u and $[T_1, T_2]$ is the delivery period (with $T_2 - T_1$ representing exactly one calendar month) of the electricity future calculated from the actual time t and the time to maturity u. Hence, two new parameters $\theta_Q := \mathbb{E}_Q [Z(1)]$ (expectation under a risk-neutral measure Q) and C (shift parameter for futures) are used here.

Finally, the CARMA(2, 1) process can be described by its state-space representation

$$Y_t = \boldsymbol{b}^T\boldsymbol{X_t}$$
$$\boldsymbol{dX_t} = A\boldsymbol{X_t}\,dt + \boldsymbol{e_p}\,dL_t$$

where

$$\boldsymbol{b}^T = (b_0, 1),\ \boldsymbol{e}_2^T = (0, 1) \text{ and } A = \begin{pmatrix} 0 & 1 \\ -a_2 & -a_1 \end{pmatrix}.$$

Benth *et al.* (2014) have chosen an α-stable Lévy process L to capture the extreme spikes in the electricity spot prices, which decrease always relatively quickly again after occurrence. Using various model verification techniques, they have shown that the model describes the electricity market very well.

However, fitting the model to electricity spot prices by the method developed in Müller and Seibert (2019), and conducting a running window analysis (cf. Fig. 2) with window length 2 years (shifted by 13 weeks each), one can see that the parameters α and β are most likely not constant over time. Whereas the stability parameter α seems to increase signicantly, the skewness parameter β turns from positive to negative values. This is in accordance with the observation that formerly the spikes were mainly upwards, whereas nowadays the spikes are mostly downwards.

Hence, we replace the α-stable process L in the model by a NASA process as introduced in Section 2. For describing the timely variation in the parameter β we use

$$\beta(t) = \beta_{\text{start}} + st/T,$$

with $T = 2922$ (representing the number of observations), whereas we use two dierent approaches for the parameter α:

Exponentially shaped:

$$\alpha(t) = 2 - (2 - \alpha_{\text{start}}) \exp(-rt/T), \tag{3}$$

Linearly shaped:

$$\alpha(t) = \alpha_{\text{start}} + \tilde{r}t/T. \tag{4}$$

In both setups we need three CARMA parameters (b_0, a_1, a_2) and 5 parameters for the driving NASA process $(\alpha_{\text{start}}, r(\tilde{r}, \text{resp.}), \beta_{\text{start}}, s, \gamma)$.

Müller and Seibert (2019) give explicit expressions for the likelihood of the model. Since the CARMA part is estimated using the likelihood from a corresponding ARMA process (see also Benth *et al.* (2014)), we can easily replace this likelihood by the (approximative) likelihood for the NASA process, where we assume that, locally on very small time intervals, α-stable increments can be used instead of the (true) NASA increments. This seems justied since in our setup the parameters α and β change only very slowly over time (relatively to the total number of observations).

Using the Bayesian framework we will be able to decide whether one of these two possibilities is superior for modelling the data set at hand, and most importantly, whether we need an additive process at all. However, before we apply the model to electricity price data, we conduct some simulations in order to assess the quality of the Bayesian estimates for all model parameters when the driving α-stable Lévy process in the electricity price model is replaced by a NASA process. In particular, we are interested how well the NASA parameters can be reconstructed from this data in such a complex model.

4.2 Simulations

We now simulate 100 data sets from the full spot price model with 3000 observations, and assume that $\alpha(t)$ follows Equation (3). We have chosen the true parameters to be realistic in view of electricity price data as analysed later. The model parameters are fitted to the simulated data sets using the Markov chain Monte Carlo method by Müller and Seibert (2019) adapted to the NASA driven CARMA process as described above.

Figure 3 shows density estimates for the 100 posterior mean estimates each together with the corresponding true values (vertical dashed lines) and the means over the 100 posterior mean estimates (vertical solid lines). Obviously, we can estimate all parameters quite precisely on average, since the two vertical lines in each subplot are always close to each other and even overlap in a few cases. The variation around the true values can be reduced by using more observations.

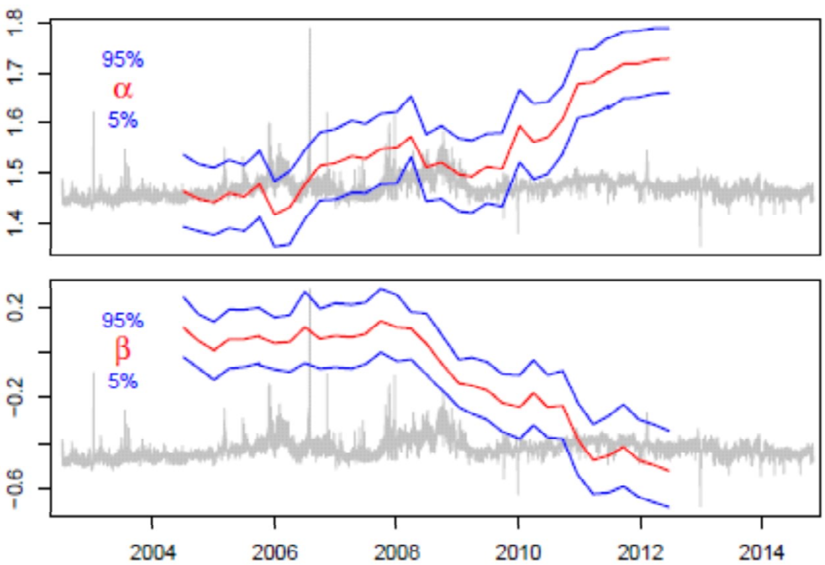

Figure 2: Rolling window analysis for electricity spot price data: posterior mean estimates together with 5% and 95% quantiles. Original electricity spot prices in the background.

4.3 Empirical Results

In this section we analyse electricity base spot prices from the EEX for the years 2002 to 2014 using the two setups described by Equations (3) and (4).

Table 2 reports posterior mean estimates for the 29 model parameters, together with 95% credibility intervals, for both model setups. As expected, the NIG, CARMA, seasonality and futures parameters are only slightly affected if one switches from the exponentially shaped to the linearly shaped function α. Most interestingly for our analysis and the motivation of this paper is, that the 95% credibility intervals for r, $\tilde{r}$, and s each do not contain 0. This means that the functions α and β and are indeed not constant over time, and replacing the Lévy process L by a NASA process is statistically a clear improvement for describing electricity data. To compare the two setups, we use the AIC criterion evaluated in each iteration of the MCMC procedure. The mean and the 2.5% and 97.5% quantiles are also reported in Table 2. From the means one can see that the linear setup is slightly better, although there is statistically no significant difference between the two setups (for this data set), as can be concluded from the corresponding quantiles.

Figure 4 shows estimated posterior densities for the four parameters in the driving NASA process for both setups. Figure 5 displays the estimated stability function α and skewness function β based on the posterior mean estimates for both the exponential (dashed lines) and the linear setup (solid lines). Due to the small positive rate parameter r there is hardly any difference between the two approaches for this data set. Moreover, the curves show again that the assumption of constant values for α and β would be unrealistic. Hence, using an additive process as NASA contributes to a more realistic modelling of electricity prices and, in consequence, to a more accurate risk management for both producers and consumers of energy.

5. Summary

In this paper we defined the class of NASA processes, a class of additive processes, which behave locally similar to α-stable processes, but allow the stable parameters to vary over time. Under mild conditions, the NASA processes are semimartingales. As we have seen in simulation studies, we can estimate NASA processes with smoothly changing parameters - the case which seems most relevant for practical applications.

The paper opens a door to handle real-life appliations with nonstationary behavior using a generalization of α-stable processes. The basic idea is quite general and could be used to define other additive processes which mimic locally the behavior of any corresponding Lévy process. Since we a faced with rapidly changing structures in many areas of business and economics, our approach may help to describe phenomena more realistically and, therefore, to improve risk management.

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Table 2: Parameter estimates for EEX data, together with AIC, based on 20000 MCMC iterations.

	α exponentially shaped			*α linearly shaped*		
Parameter	*Post. mean*	*95% cred.*	*Interval*	*Post. mean*	*95% cred.*	*Interval*
α_{start}	1.44661	.3442	1.5453	1.4274	1.3347	1.5156
α_{start}	0.0606	–0.0332	0.1664	0.0609	–0.0326	0.1406
r	0.4080	0.0762	0.7211			
$\tilde{r}$				0.2345	0.0664	0.4127
s	–0.3769	–0.5560	–0.2145	–0.3613	–0.5163	–0.1948
γ	4.1884	3.9967	4.4027	4.1654	4.0164	4.3605
a_1	0.5637	0.4534	0.7524	0.5403	0.4532	0.7149
a_2	0.0110	0.0018	0.0350	0.0084	0.0009	0.0300
b_0	0.0660	0.0226	0.1650	0.0559	0.0195	0.1492
α_{NIG}	0.7951	0.6844	0.9296	0.7782	0.6713	0.8923
β_{NIG}	0.0179	–0.0399	0.0733	0.0210	–0.0364	0.0758
δ_{NIG}	0.3254	0.3012	0.3503	0.3223	0.2995	0.3439
θ_{summer}	–2.9481	–4.4193	–1.6999	–3.0602	–4.3955	–1.8141
α_{Dec}	–8.9864	–11.0709	–6.8850	–8.9926	–11.2761	–6.7782
α_{Jan}	–8.2983	–10.7545	–5.6565	–8.7383	–11.1939	–5.7888
c_1	–5.1713	–5.6449	–4.7865	–5.1649	–5.6067	–4.7264
s_1	0.0760	–0.2667	0.4266	0.0724	–0.3095	0.4738
c_2	0.8089	0.4266	1.2259	0.7921	0.41161	1.2217
s_2	0.7700	0.3149	1.2261	0.7553	0.3192	1.2043
θ_{trend}	–0.0029	–0.0142	0.0086	–0.0034	–0.0154	0.0096
α_{Tu}	1.6707	1.2502	2.0754	1.6286	1.1970	2.1042
θ_{We}	1.6509	1.1135	2.1795	1.5683	1.0220	2.1454
α_{Th}	1.0675	0.5382	1.6019	1.0309	0.5078	1.6159
α_{Fr}	–0.7424	–1.2842	–0.1220	–0.7516	–1.3801	–0.1164
θ_{Sa}	–8.2988	–8.8047	–7.7554	–8.3382	–8.8725	–7.7202
θ_{Su}	–14.7749	–15.2420	–14.2885	–14.7619	–15.1650	–14.2581
θ_{ho}	–10.8910	–11.9640	–9.8327	–10.9139	–11.9113	–9.8733
θ_0	20.3262	17.1624	22.8040	19.2654	15.1151	22.9251
θ_Q	0.0147	0.0026	0.0262	0.0152	0.0020	0.0281
C	5.6766	3.2168	8.8374	6.7118	3.2395	10.9649
AIC	34054.72	34042.61	34071.03	34052.92	34038.16	34071.99

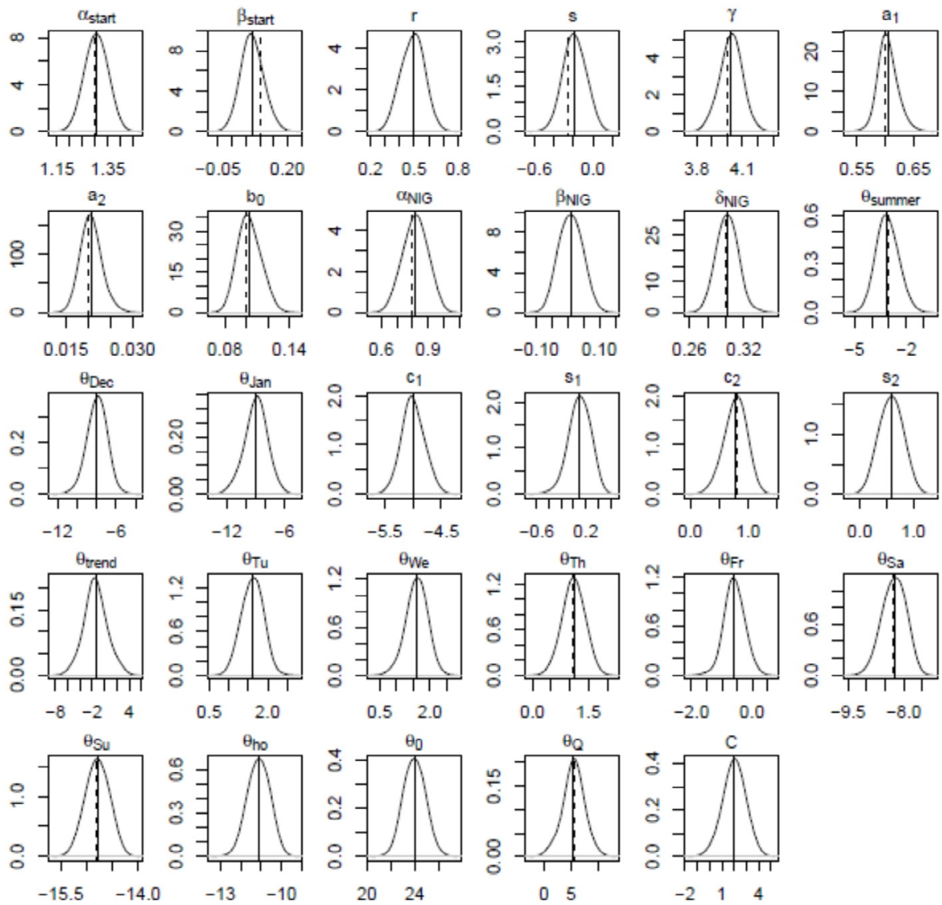

Figure 3: Densities of posterior mean estimates accross 100 simulated data sets. Vertical dashed lines indicate the true values used for simulation of the data sets, vertical solid lines indicate the means over the 100 posterior mean estimates each.

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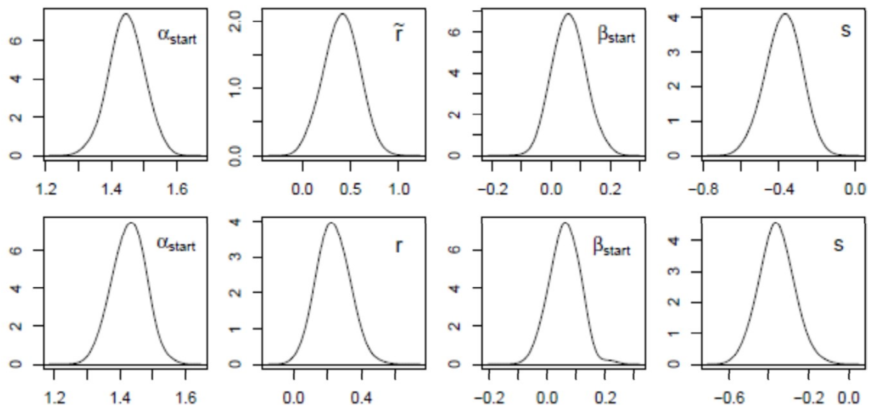

Figure 4: Top row: estimated posterior densities for α_{start}, r, β_{start}, and s in the exponential setup, cf. Eq. (3). Bottom row: estimated posterior densities for α_{start}, $\tilde{r}$, β_{start}, and s in the linear setup, cf. Eq. (4).

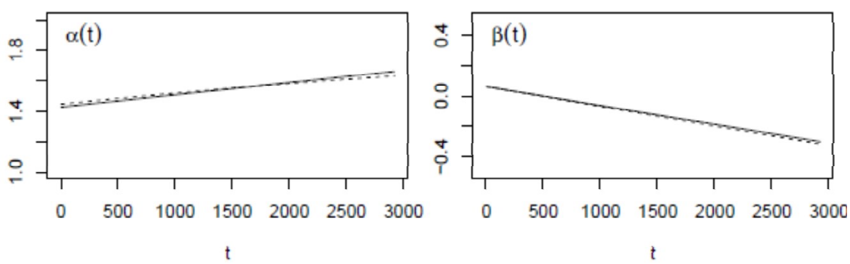

Figure 5: Estimated functions $\alpha(t)$ and $\beta(t)$ based on the posterior mean estimates for both the exponential (dashed lines) and the linear setup (solid lines).

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